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Off-diagonal helicity density matrix elements for vector mesons produced at LEP

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Abstract. Final state $q\bar{q}$ interactions may give origin to non-zero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons produced in e^+e^- annihilations, as confirmed by recent OPAL data on ϕ and D^*'s. Predictions are given for $\rho_{1,-1}$ of several mesons produced at large z and small p_T, *i.e.* collinear with the parent jet; the values obtained for ϕ and D^* are in agreement with data.

1 Introduction

The spin properties of hadrons inclusively produced in high energy interactions are related to the fundamental properties of quarks and gluons and to their elementary interactions in a much more subtle way than unpolarized quantities; the usual hadronization models – successful in predicting unpolarized cross-sections – may not be adequate to describe spin effects, say the fragmentation of a polarized quark.

In [1] and [2] it was pointed out how the final state interactions between the q and $\bar{q}$ produced in e^+e^- annihilations – usually neglected, but indeed necessary – might give origin to non zero spin observables which would otherwise be forced to vanish. The off-diagonal matrix element $\rho_{1,-1}$ of vector mesons may be sizeably different from zero [1] due to a coherent fragmentation process which takes into account $q\bar{q}$ interactions; the incoherent fragmentation of a single independent quark leads to zero values for such off-diagonal elements. The same situation is not true for spin 1/2 baryons, for which the coherent fragmentation process only induces corrections which vanish in the limit of small transverse momentum, p_T, of the quark inside the jet [2]. Both predictions, a non zero value of $\rho_{1,-1}$ for D^* and possibly ϕ particles [3], and a value $\rho_{+-} \simeq (p_T/z\sqrt{s})$ for Λ (*i.e.*, its transverse polarization) [4] have recently been confirmed experimentally.

We consider here in greater details the coherent fragmentation process of $q\bar{q}$ produced at LEP, where the quarks are strongly polarized; we are actually able to give predictions for $\rho_{1,-1}$ of several vector mesons V provided they are produced in two jet events, carry a large momentum or energy fraction $z = 2E_V/\sqrt{s}$, and have a small transverse momentum p_T inside the jet. Our estimates are in agreement with the existing data and are crucially related both to the presence of final state interactions and to the Standard Model couplings of the elementary $e^-e^+ \to q\bar{q}$ interaction.

In the next section we review the formalism to compute the helicity density matrix of a hadron produced in $e^-e^+ \to q\bar{q} \to h + X$ processes and give analytical expressions for the nondiagonal matrix element $\rho_{1,-1}$ in case of final spin 1 hadrons; in Sect. 3 we obtain numerical estimates and in Sect. 4 we make some further comments and conclusions.

2 $\rho_{1,-1}(V)$ in the process $e^-e^+ \to q\bar{q} \to V + X$

The helicity density matrix of a hadron h inclusively produced in the two jet event $e^-e^+ \to q\bar{q} \to h + X$ can be written as [1,2]

$$\rho_{\lambda_h\lambda'_h}(h) = \frac{1}{N_h} \sum_{q,X,\lambda_X,\lambda_q,\lambda_{\bar{q}},\lambda'_q,\lambda'_{\bar{q}}} D_{\lambda_h\lambda_X;\lambda_q,\lambda_{\bar{q}}} \times \rho_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) \; D^*_{\lambda'_h\lambda_X;\lambda'_q,\lambda'_{\bar{q}}}\,, \tag{1}$$

where $\rho_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q})$ is the helicity density matrix of the $q\bar{q}$ state created in the annihilation of the unpolarized e^+ and e^-,

$$\rho_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q}) = \frac{1}{4N_{q\bar{q}}} \sum_{\lambda_-,\lambda_+} M_{\lambda_q\lambda_{\bar{q}};\lambda_-\lambda_+} \times M^*_{\lambda'_q\lambda'_{\bar{q}};\lambda_-\lambda_+}\,. \tag{2}$$

The M's are the helicity amplitudes for the $e^-e^+ \to q\bar{q}$ process and the D's are the fragmentation amplitudes, *i.e.*

the helicity amplitudes for the process $q\bar{q} \to h + X$; the $\sum_{X,\lambda_X}$ stands for the phase space integration and the sum over spins of all the unobserved particles, grouped into a state X. The normalization factors N_h and $N_{q\bar{q}}$ are given by

$$N_h = \sum_{q,X;\lambda_h,\lambda_X,\lambda_q,\lambda_{\bar{q}},\lambda'_q,\lambda'_{\bar{q}}} D_{\lambda_h\lambda_X;\lambda_q,\lambda_{\bar{q}}} \times \rho_{\lambda_q,\lambda_{\bar{q}};\lambda'_q,\lambda'_{\bar{q}}}(q\bar{q})\; D^*_{\lambda_h\lambda_X;\lambda'_q,\lambda'_{\bar{q}}} = \sum_q D_q^h\,, \tag{3}$$

where D_q^h is the usual fragmentation function of quark q into hadron h [see also comment after (22)], and

$$N_{q\bar{q}} = \frac{1}{4}\sum_{\lambda_q,\lambda_{\bar{q}};\lambda_-,\lambda_+} |M_{\lambda_q\lambda_{\bar{q}};\lambda_-\lambda_+}|^2\,. \tag{4}$$

The center of mass helicity amplitudes for the $e^-e^+ \to q\bar{q}$ process can be computed in the Standard Model and are given by

$$\begin{aligned} M_{\lambda_q\lambda_{\bar{q}};\lambda_-\lambda_+}(s,\theta) =\; & e^2\;\delta_{\lambda_-,-\lambda_+}\,\delta_{\lambda_q,-\lambda_{\bar{q}}}\times \\ \times \Bigg\{ & \left[e_q - g_Z(s)\, g_V^l\, g_V^q\right](1+4\lambda_-\lambda_q\cos\theta) \\ & + g_Z(s)\Big[2\, g_V^l\, g_A^q\,(\lambda_-\cos\theta+\lambda_q) \\ & + 2\, g_A^l\, g_V^q\,(\lambda_- + \lambda_q\cos\theta) - g_A^l\, g_A^q\,(\cos\theta + 4\lambda_-\lambda_q)\Big]\Bigg\}\,, \end{aligned} \tag{5}$$

where $\sqrt{s}$ is the total e^+e^- c.m. energy, θ the q production angle (*i.e.* the angle between the incoming e^- and the outgoing q) and e_q is the quark charge. Lepton and quark masses have been neglected with respect to their energies and we report here for convenience the Standard Model coupling constants:

$$\begin{aligned} g_V^l &= -\frac{1}{2} + 2\sin^2\theta_W & g_A^l &= -\frac{1}{2} \\ g_V^{u,c,t} &= \frac{1}{2} - \frac{4}{3}\sin^2\theta_W & g_A^{u,c,t} &= \frac{1}{2} \\ g_V^{d,s,b} &= -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W & g_A^{d,s,b} &= -\frac{1}{2} \\ g_Z(s) &= \frac{1}{4\sin^2\theta_W\cos^2\theta_W}\;\frac{s}{(s-M_Z^2)+iM_Z\Gamma_Z}\,. \end{aligned} \tag{6}$$

From (2), (4) and (5) one finds the explicit expressions of the only non zero elements of $\rho(q\bar{q})$:

$$\rho_{+-;+-}(q\bar{q}) = 1 - \rho_{-+;-+}(q\bar{q}) = \frac{a'_q(s)\;(1+\cos^2\theta) - b'_q(s)\cos\theta}{\mu_q(s)\;(1+\cos^2\theta) + \eta_q(s)\cos\theta} \tag{7}$$

$$\rho_{+-;-+}(q\bar{q}) = \rho^*_{-+;+-}(q\bar{q}) = \frac{[a_q(s) - ib_q(s)]\;\sin^2\theta}{\mu_q(s)\;(1+\cos^2\theta) + \eta_q(s)\cos\theta} \tag{8}$$

where $+,-$ stand for helicity $+1/2$ and $-1/2$ and where, for an arbitrary total energy $\sqrt{s}$,

$$\begin{aligned} a'_q(s) &= e_q^2 + |g_Z(s)|^2\;(g_V - g_A)_q^2\;(g_V^2 + g_A^2)_l \\ &\quad -2e_q\;\mathrm{Re}[g_Z(s)]\; g_V^l\,(g_V - g_A)_q \\ b'_q(s) &= 4g_A^l\,(g_V - g_A)_q\;\left[\;|g_Z(s)|^2\; g_V^l\,(g_V - g_A)_q\right. \\ &\quad \left. -e_q\;\mathrm{Re}[g_Z(s)]\right] \end{aligned}$$

$$\begin{aligned} a_q(s) &= e_q^2 + |g_Z(s)|^2\;(g_V^2 - g_A^2)_q\;(g_V^2 + g_A^2)_l \\ &\quad -2e_q\; g_V^l g_V^q\;\mathrm{Re}[g_Z(s)] \\ b_q(s) &= -2e_q\; g_V^l g_A^q\;\mathrm{Im}[g_Z(s)] \end{aligned} \tag{9}$$

$$\begin{aligned} \mu_q(s) &= 2\left[\;e_q^2 + |g_Z(s)|^2\;(g_V^2 + g_A^2)_l(g_V^2 + g_A^2)_q\right. \\ &\quad \left. -2e_q\; g_V^l g_V^q\;\mathrm{Re}[g_Z(s)]\right] \\ \eta_q(s) &= 8g_A^l g_A^q\;\left[\;2\;|g_Z(s)|^2\; g_V^l g_V^q - e_q\;\mathrm{Re}[g_Z(s)]\right] \end{aligned}$$

which at $\sqrt{s} = M_Z$ read

$$\begin{aligned} a'_q &= e_q^2 + \zeta^2\,(g_V - g_A)_q^2\;(g_V^2 + g_A^2)_l \\ b'_q &= 4\zeta^2\,(g_A g_V)_l\;(g_V - g_A)_q^2 \\ a_q &= e_q^2 + \zeta^2\;(g_V^2 - g_A^2)_q\;(g_V^2 + g_A^2)_l \\ b_q &= 2e_q\;\zeta\, g_V^l g_A^q \\ \mu_q &= 2[\;e_q^2 + \zeta^2\;(g_V^2 + g_A^2)_l\;(g_V^2 + g_A^2)_q] \\ \eta_q &= 16\zeta^2\,(g_A g_V)_l\;(g_A g_V)_q \\ \zeta &= \frac{M_Z}{4\;\Gamma_Z\;\sin^2\theta_W\;\cos^2\theta_W}\,. \end{aligned} \tag{10}$$

Equations (7) and (8) hold for the production of a quark with flavour q at a c.m. angle θ, defined as the angle between the incoming negative lepton and the outgoing quark; in the $p_T \to 0$ limit this is the same angle as the production angle of the observed hadron h. However, h can be produced also in the fragmentation of an antiquark $\bar{q}$ and the $\sum_q$ in (1) and (3) takes into account also this possibility ($q = u, d, s, c, b, \bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}$): the helicity density matrix $\rho(\bar{q}q)$ for the production of an antiquark at the angle θ can be obtained from $\rho(q\bar{q})$ with the simple replacements:

$$\begin{aligned} \rho_{+-;+-}(\bar{q}q,\theta) &= \rho_{-+;-+}(q\bar{q},\pi-\theta) \\ \rho_{+-;-+}(\bar{q}q,\theta) &= \rho^*_{+-;-+}(q\bar{q},\pi-\theta)\,. \end{aligned} \tag{11}$$

The expressions (8), (9) and (10) are exact and contain both electromagnetic and weak interaction contributions. However, at LEP energy $\sqrt{s} = M_Z$, the weak contribution dominates, $\zeta \gg 1$ in (10); if one also takes into account that η_q is depressed by the small value of g_V^l a simple approximate and useful formula for $\rho_{+-;-+}$ is given by [for an exact value at $\sqrt{s} = M_Z$ see (40)]

$$\rho^Z_{+-;-+}(q\bar{q}) \simeq \frac{1}{2}\,\frac{(g_V^2 - g_A^2)_q}{(g_V^2 + g_A^2)_q}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,. \tag{12}$$

Equation (12) clearly shows the θ dependence of $\rho_{+-;-+}$. This approximate expression is the same both for $\rho(q\bar{q})$ and $\rho(\bar{q}q)$. In the case of pure electromagnetic interactions ($\sqrt{s} \ll M_Z$) one has exactly:

$$\rho^{\gamma}_{+-;-+}(q\bar{q}) = \frac{1}{2}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,. \qquad (13)$$

Notice that (12) and (13) have the same angular dependence, but a different sign for the coefficient in front, which is negative for the Z contribution [see (6)].

By using the above equations for $\rho(q\bar{q})$ [and $\rho(\bar{q}q)$] into (1) one obtains the most general expression of $\rho(h)$ in terms of the $q\bar{q}$ spin state and the unknown fragmentation amplitudes [2]. Such expression can be greatly simplified if one considers the production of hadrons almost collinear with the parent jet: the $q\bar{q} \to h+X$ fragmentation is then essentially a c.m. forward process and the unknown D amplitudes must satisfy the angular momentum conservation relation [5]

$$D_{\lambda_h\lambda_X;\lambda_q,\lambda_{\bar{q}}}\,D^*_{\lambda'_h\lambda_X;\lambda'_q,\lambda'_{\bar{q}}} \sim \left(\sin\frac{\theta_h}{2}\right)^{|\lambda_h-\lambda_X-\lambda_q+\lambda_{\bar{q}}|+|\lambda'_h-\lambda_X-\lambda'_q+\lambda'_{\bar{q}}|}\,, \qquad (14)$$

where θ_h is the angle between the hadron momentum, $\boldsymbol{h} = z\boldsymbol{q} + \boldsymbol{p}_T$, and the quark momentum $\boldsymbol{q}$, that is

$$\sin\theta_h \simeq \frac{2p_T}{z\sqrt{s}}\,. \qquad (15)$$

The bilinear combinations of fragmentation amplitudes contributing to $\rho(h)$ are then not suppressed by powers of $(p_T/(z\sqrt{s}))$ only if the exponents in (14) are zero; which yields

$$\lambda_X = \lambda_h - (\lambda_q - \lambda_{\bar{q}}) = \lambda'_h - (\lambda'_q - \lambda'_{\bar{q}})\,. \qquad (16)$$

In the $p_T \to 0$ limit one has then the simple result for the nondiagonal density matrix elements of spin 1 mesons [1,2]:

$$\mathrm{Re}[\rho_{1,-1}(V)] = \frac{1}{N_h}\sum_{X,q} D_{10;+-}\,D^*_{-10;-+}\,\mathrm{Re}[\rho_{+-;-+}(q\bar{q})] \qquad (17)$$

$$\mathrm{Im}[\rho_{1,-1}(V)] = \frac{1}{N_h}\sum_{X,q} D_{10;+-}\,D^*_{-10;-+}\,\mathrm{Im}[\rho_{+-;-+}(q\bar{q})] \qquad (18)$$

with

$$N_h = \sum_q D^h_q = \sum_{q,X;\lambda_h,\lambda_X}\Big[|D_{\lambda_h\lambda_X;+-}|^2\,\rho_{+-;+-}(q\bar{q}) + |D_{\lambda_h\lambda_X;-+}|^2\,\rho_{-+;-+}(q\bar{q})\Big]\,. \qquad (19)$$

Equations (17) and (18) explicitly show that the coherent quark fragmentation allows non zero off-diagonal helicity density matrix elements which, for vector mesons, survive also in the small p_T limit; the other off-diagonal matrix elements for spin 1 particles and all off-diagonal matrix elements for spin 1/2 particles are bound, via (14), to vanish at small $p_T/\sqrt{s}$ values [1,2]. Recent experimental data have confirmed both the non-zero value of $\rho_{1,-1}(D^*)$ [3] and the small value of $\rho_{+-}(\Lambda)$ [4].

In the next section we give numerical estimates of $\rho_{1,-1}$ for several vector mesons, exploiting (17) and the fact that, at least for valence quark contributions, the dependence on the fragmentation amplitudes either cancels out or can be expressed in terms of other measured quantities.

3 Numerical estimates of $\rho_{1,-1}(V)$ at $\sqrt{s} = M_Z$

Let us consider (17)-(19). Despite our ignorance of the fragmentation amplitudes we see that in the $p_T \to 0$ limit, due to (14) and (16), only few of them give a leading contribution; moreover, the fragmentation is a parity conserving forward process, so that the fragmentation amplitudes must satisfy the relationship [5]

$$D_{-\lambda_h-\lambda_X;-+} = (-1)^{S_h+S_X+\lambda_h-\lambda_X}\,D_{\lambda_h\lambda_X;+-}\,, \qquad (20)$$

where S_h and S_X are respectively the spin of hadron h and of the unobserved system X (the intrinsic parities of the initial and final states must be the same). In particular (20) for spin 1 hadrons yields

$$D_{-10;-+} = (-1)^{S_X}\,D_{10;+-}\,. \qquad (21)$$

Notice that the parity relationship (20) and (7) allow to write:

$$D^h_q = \sum_{X;\lambda_h,\lambda_X}|D_{\lambda_h\lambda_X;+-}|^2\,, \qquad (22)$$

which is the fragmentation function of quark q into hadron h, whose spin is not observed; such fragmentation function is independent of the quark polarization, described by $\rho(q\bar{q})$. Instead, the fragmentation functions of a polarized quark q into a hadron h with helicity λ_h are given by:

$$D^{h,\lambda_h}_q = \sum_{X;\lambda_X}\Big[|D_{\lambda_h\lambda_X;+-}|^2\,\rho_{+-;+-}(q\bar{q}) + |D_{\lambda_h\lambda_X;-+}|^2\,\rho_{-+;-+}(q\bar{q})\Big] = D^{h,\lambda_h}_{q,+}\,\rho_{+-;+-}(q\bar{q}) + D^{h,\lambda_h}_{q,-}\,\rho_{-+;-+}(q\bar{q})\,, \qquad (23)$$

which is consistent with $\sum_{\lambda_h} D^{h,\lambda_h}_q = D^h_q$ and where $D^{h,\lambda_h}_{q,\lambda_q}$ is the fragmentation function of quark q with helicity λ_q into hadron h with helicity λ_h; $\rho_{\lambda_q,-\lambda_q;\lambda_q,-\lambda_q}(q\bar{q})$ is the probability for q to have helicity λ_q.

Taking into account (16) and (20) the above fragmentation functions read:

$$D^h_q = \sum_X\Big[|D_{10;+-}|^2 + |D_{0-1;+-}|^2 + |D_{-1-2;+-}|^2\Big]$$

$$= D_{q,+}^{h,1} + D_{q,+}^{h,0} + D_{q,+}^{h,-1} \tag{24}$$

$$D_q^{h,1} = \sum_X \Big[|D_{10;+-}|^2 \, \rho_{+-;+-}(q\bar{q}) + |D_{12;-+}|^2 \, \rho_{-+;-+}(q\bar{q}) \Big]$$

$$= D_{q,+}^{h,1} \, \rho_{+-;+-}(q\bar{q}) + D_{q,-}^{h,1} \, \rho_{-+;-+}(q\bar{q}) \tag{25}$$

$$D_q^{h,0} = \sum_X |D_{0-1;+-}|^2 = D_{q,+}^{h,0} \tag{26}$$

$$D_q^{h,-1} = \sum_X \Big[|D_{12;-+}|^2 \, \rho_{+-;+-}(q\bar{q}) + |D_{10;+-}|^2 \, \rho_{-+;-+}(q\bar{q}) \Big]$$

$$= D_{q,-}^{h,1} \, \rho_{+-;+-}(q\bar{q}) + D_{q,+}^{h,1} \, \rho_{-+;-+}(q\bar{q}) \,. \tag{27}$$

We now assume that, at least for valence quarks:

$$D_{q,-}^{h,1} = D_{q,+}^{h,-1} = 0 \tag{28}$$

$$D_{q,+}^{h,0} = \alpha_q^V \, D_{q,+}^{h,1} \,. \tag{29}$$

The first of these assumptions simply means that quarks with helicity 1/2 (−1/2) cannot fragment into vector mesons with helicity −1 (+1). This is true for valence quarks assuming vector meson wave functions with no orbital angular momentum, like in $SU(6)$. The second assumption is also true in $SU(6)$ with $\alpha_q^V = 1/2$ for any valence q and V. Rather than taking $\alpha_q^V = 1/2$ we prefer to relate the value of α_q^V to the value of $\rho_{0,0}(V)$ which can be or has been measured. In fact, always in the $p_T \to 0$ limit, one has, from (1), (16), (20), (28) and (29):

$$\rho_{0,0}(V) = \frac{\sum_q \alpha_q^V \, D_{q,+}^{h,1}}{\sum_q (1+\alpha_q^V) \, D_{q,+}^{h,1}} \,. \tag{30}$$

If α_q^V is the same for all valence quarks in V ($\alpha_q^V = \alpha^V$) one has, for the valence quark contribution:

$$\alpha^V = \frac{\rho_{0,0}(V)}{1-\rho_{0,0}(V)} \,. \tag{31}$$

Notice that the $SU(6)$ value $\alpha_q^V = 1/2$ correspond to $\rho_{0,0} = 1/3$, that is no alignment, $A = (1/2)(3\rho_{0,0} - 1) = 0$, for the vector meson.

If we now use (21), (24), (28), (29) into (17) and (18) we obtain

$$\rho_{1,-1}(V) = \frac{\sum_{q,X} (-1)^{S_X} \, |D_{10;+-}|^2 \, \rho_{+-;-+}(q\bar{q})}{\sum_{q,X} (1+\alpha_q^V) \, |D_{10;+-}|^2} \,. \tag{32}$$

The numerator in the above equation depends on the squared amplitude $|D_{10;+-}|^2$ for the $q\bar{q} \to V + X$ forward fragmentation process and on S_X. The $q\bar{q}$ state is such that $J = J_z = 1$; the final undetected system X must then have $\lambda_X = 0$ with $S_X = 0, 1$ or 2, the only states which can combine with the $S_h = \lambda_h = 1$ vector meson state to give a VX spin state with $J = J_z = 1$. On a simple statistical basis these 3 possible states have respectively relative probabilities 1, 1/6 and 1/30. One can then conclude that the $S_X = 0$ state dominates and approximate the above equation (32) with

$$\rho_{1,-1}(V) \simeq \frac{\sum_q D_{q,+}^{V,1} \, \rho_{+-;-+}(q\bar{q})}{\sum_q (1+\alpha_q^V) \, D_{q,+}^{V,1}} \,. \tag{33}$$

The actual value (32) should only be slightly smaller, due to some contribution from $S_X = 1$.

Again, if only one flavour contributes or if we can assume that α_q^V does not depend on the valence quark flavour, (31) further simplifies (33) to

$$\rho_{1,-1}(V) \simeq [1-\rho_{0,0}(V)] \, \frac{\sum_q D_{q,+}^{V,1} \, \rho_{+-;-+}(q\bar{q})}{\sum_q D_{q,+}^{V,1}} \,. \tag{34}$$

We shall now consider some specific cases in which we expect (34) to hold; let us remind once more that our conclusions apply to spin 1 vector mesons produced in $e^-e^+ \to q\bar{q} \to V+X$ processes in the limit of small p_T and large z, *i.e.*, to vector mesons produced in two jet events ($e^-e^+ \to q\bar{q}$) and collinear with one of them ($p_T = 0$), which is the jet generated by a quark which is a valence quark for the observed vector meson (large z). These conditions should be met more easily in the production of heavy vector mesons.

Let us then start from the cases $V = B^{*\pm,0}$, $D^{*\pm,0}$. In such a case one can safely assume that the fragmenting quark is the heavy one so that (34) applies and one has:

$$\begin{aligned} \rho_{1,-1}(B^{*+}) &\simeq [1-\rho_{0,0}(B^{*+})] \, \rho_{+-;-+}(\bar{b}b) \\ \rho_{1,-1}(B^{*-}) &\simeq [1-\rho_{0,0}(B^{*-})] \, \rho_{+-;-+}(b\bar{b}) \\ \rho_{1,-1}(B^{*0}) &\simeq [1-\rho_{0,0}(B^{*0})] \, \rho_{+-;-+}(\bar{b}b) \end{aligned} \tag{35}$$

$$\begin{aligned} \rho_{1,-1}(D^{*+}) &\simeq [1-\rho_{0,0}(D^{*+})] \, \rho_{+-;-+}(c\bar{c}) \\ \rho_{1,-1}(D^{*-}) &\simeq [1-\rho_{0,0}(D^{*-})] \, \rho_{+-;-+}(\bar{c}c) \\ \rho_{1,-1}(D^{*0}) &\simeq [1-\rho_{0,0}(D^{*0})] \, \rho_{+-;-+}(c\bar{c}) \end{aligned} \tag{36}$$

Similarly one obtains:

$$\rho_{1,-1}(\phi) \simeq \frac{1}{2} \, [1-\rho_{0,0}(\phi)] \, [\rho_{+-;-+}(s\bar{s}) + \rho_{+-;-+}(\bar{s}s)] \tag{37}$$

where we have assumed $D_{s,+}^{\phi,1} = D_{\bar{s},+}^{\phi,1}$, as it should be.

For ρ's, assuming all valence quark fragmentation functions to be the same, one has

$$\begin{aligned} \rho_{1,-1}(\rho^+) &\simeq \frac{1}{2} \, [1-\rho_{0,0}(\rho^+)] \, [\rho_{+-;-+}(u\bar{u}) + \rho_{+-;-+}(\bar{d}d)] \\ \rho_{1,-1}(\rho^0) &\simeq \frac{1}{4} \, [1-\rho_{0,0}(\rho^0)] \, [\rho_{+-;-+}(u\bar{u}) + \rho_{+-;-+}(d\bar{d}) \\ &\quad + \rho_{+-;-+}(\bar{u}u) + \rho_{+-;-+}(\bar{d}d)] \\ \rho_{1,-1}(\rho^-) &\simeq \frac{1}{2} \, [1-\rho_{0,0}(\rho^-)] \, [\rho_{+-;-+}(d\bar{d}) + \rho_{+-;-+}(\bar{u}u)] \,. \end{aligned} \tag{38}$$

The assumption that all valence quark fragmentation functions are the same is very natural for ρ's, but it might be weaker for K^* mesons; if nevertheless we assume that, at least at large z, $D^{K^{*+},1}_{\bar{s},+} = D^{K^{*+},1}_{u,+}$, and similarly for K^{*0} and K^{*-} , we have

$$\begin{aligned}
\rho_{1,-1}(K^{*+}) &\simeq \frac{1}{2}[1-\rho_{0,0}(K^{*+})] \\
&\quad \times[\rho_{+-;-+}(u\bar{u}) + \rho_{+-;-+}(\bar{s}s)] \\
\rho_{1,-1}(K^{*0}) &\simeq \frac{1}{2}[1-\rho_{0,0}(K^{*0})] \\
&\quad \times[\rho_{+-;-+}(d\bar{d}) + \rho_{+-;-+}(\bar{s}s)] \\
\rho_{1,-1}(K^{*-}) &\simeq \frac{1}{2}[1-\rho_{0,0}(K^{*-})] \\
&\quad \times[\rho_{+-;-+}(\bar{u}u) + \rho_{+-;-+}(s\bar{s})]\,.
\end{aligned} \tag{39}$$

A predominant contribution of the s quark would instead lead to results similar to those found for B^*.

Equations (35)-(39) show how the value of $\rho_{1,-1}(V)$ are simply related to the off-diagonal matrix element $\rho_{+-;-+}(q\bar{q})$ of the $q\bar{q}$ pair created in the elementary $e^-e^+ \to q\bar{q}$ process; such off-diagonal elements would not appear in the incoherent independent fragmentation of a single quark, yielding $\rho_{1,-1}(V) = 0$.

We can now make numerical predictions by inserting into the above equations the explicit values of $\rho_{+-;-+}(q\bar{q})$ at $\sqrt{s} = M_z$, (8), (10) and (6) with $\sin^2\theta_W = 0.2237$, $M_z = 91.19$ GeV, $\Gamma_z = 2.50$ GeV [6]:

$$\begin{aligned}
\rho_{+-;-+}(u\bar{u}) &= -0.36\,(1-0.013\,i) \\
&\quad \times\frac{\sin^2\theta}{(1+\cos^2\theta)+0.29\cos\theta} \\
\rho_{+-;-+}(\bar{u}u) &= -0.36\,(1+0.013\,i) \\
&\quad \times\frac{\sin^2\theta}{(1+\cos^2\theta)-0.29\cos\theta} \\
\rho_{+-;-+}(d\bar{d}) &= -0.17\,(1-0.010\,i) \\
&\quad \times\frac{\sin^2\theta}{(1+\cos^2\theta)+0.39\cos\theta} \\
\rho_{+-;-+}(\bar{d}d) &= -0.17\,(1+0.010\,i) \\
&\quad \times\frac{\sin^2\theta}{(1+\cos^2\theta)-0.39\cos\theta}\,.
\end{aligned} \tag{40}$$

The values for s, b and c quarks are respectively the same as for d and u.

If we instead use for simplicity the approximate expressions (12), valid at $\sqrt{s} = M_z$ and which are the same for $\rho_{+-;-+}(q\bar{q})$ and $\rho_{+-;-+}(\bar{q}q)$, we have the simple results

$$\begin{aligned}
\rho_{1,-1}(B^{*\pm,0}) &\simeq -0.170\;[1-\rho_{0,0}(B^*)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \\
&= -(0.109\pm0.015)\;\frac{\sin^2\theta}{1+\cos^2\theta}
\end{aligned} \tag{41}$$

$$\begin{aligned}
\rho_{1,-1}(D^{*\pm,0}) &\simeq -0.360\;[1-\rho_{0,0}(D^*)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \\
&= -(0.216\pm0.007)\;\frac{\sin^2\theta}{1+\cos^2\theta}
\end{aligned} \tag{42}$$

$$\begin{aligned}
\rho_{1,-1}(\phi) &\simeq -0.170\;[1-\rho_{0,0}(\phi)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \\
&= -(0.078\pm0.014)\;\frac{\sin^2\theta}{1+\cos^2\theta}
\end{aligned} \tag{43}$$

$$\rho_{1,-1}(\rho^{\pm,0}) \simeq -0.265\;[1-\rho_{0,0}(\rho)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \tag{44}$$

$$\rho_{1,-1}(K^{*\pm}) \simeq -0.265\;[1-\rho_{0,0}(K^*)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \tag{45}$$

$$\rho_{1,-1}(K^{*0}) \simeq -0.170\;[1-\rho_{0,0}(K^*)]\;\frac{\sin^2\theta}{1+\cos^2\theta} \tag{46}$$

where we have used $\rho_{0,0}(B^{*\pm,0}) = 0.36\pm0.09$, $\rho_{0,0}(D^{*\pm,0}) = 0.40\pm0.02$ and $\rho_{0,0}(\phi) = 0.54\pm0.08$ [3]; no data are available on $\rho_{0,0}(\rho)$ and $\rho_{0,0}(K^*)$. Notice that in such approximation $\rho_{1,-1}(V)$ is real and that the $\cos\theta$ term in the denominator of (8) has been neglected. This term would induce small differences between the values of $\rho_{1,-1}(B^{*+})$ [or $\rho_{1,-1}(D^{*+})$] and $\rho_{1,-1}(B^{*-})$ [or $\rho_{1,-1}(D^{*-})$]; it has much smaller effects on the values of $\rho_{1,-1}(\phi)$, $\rho_{1,-1}(\rho)$ and $\rho_{1,-1}(K^*)$.

Finally, in case one collects all meson produced at different angles in the full available θ range (say $\alpha < \theta < \pi-\alpha$, $|\cos\theta| < \cos\alpha$) an average should be taken in θ, weighting the different values of $\rho_{1,-1}(\theta)$ with the cross-section for the $e^-e^+ \to V+X$ process; this amounts essentially to weight the values of $\rho_{+-;-+}(q\bar{q};\theta)$ appearing in (35)-(39) and given in (40) or (12) with the cross-section for the $e^-e^+ \to q\bar{q}$ process, proportional to the normalization factor $N_{q\bar{q}}$ given in (4). Such an average has a simple analytical expression if one uses the approximate value (12):

$$\langle\rho_{1,-1}(B^{*\pm,0})\rangle_{[\alpha,\pi-\alpha]} \simeq -(0.109\pm0.015)\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha} \tag{47}$$

$$\langle\rho_{1,-1}(D^{*\pm,0})\rangle_{[\alpha,\pi-\alpha]} \simeq -(0.216\pm0.007)\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha} \tag{48}$$

$$\langle\rho_{1,-1}(\phi)\rangle_{[\alpha,\pi-\alpha]} \simeq -(0.078\pm0.014)\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha} \tag{49}$$

$$\langle\rho_{1,-1}(\rho^{\pm,0})\rangle_{[\alpha,\pi-\alpha]} \simeq -0.265\;[1-\rho_{0,0}(\rho)]\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha} \tag{50}$$

$$\langle\rho_{1,-1}(K^{*\pm})\rangle_{[\alpha,\pi-\alpha]} \simeq -0.265\;[1-\rho_{0,0}(K^*)]\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha} \tag{51}$$

$$\langle\rho_{1,-1}(K^{*0})\rangle_{[\alpha,\pi-\alpha]} \simeq -0.170\;[1-\rho_{0,0}(K^*)]\times\frac{3-\cos^2\alpha}{3+\cos^2\alpha}\,. \tag{52}$$

We have explicitly checked that the full expression (40) yields almost identical results [and a negligible imaginary part].

4 Comments and conclusions

We have computed, within a general factorization scheme, the off-diagonal helicity density matrix element $\rho_{1,-1}$ of vector mesons produced in $e^-e^+ \to q\bar{q} \to V + X$ annihilation processes; such element can be – and in few cases has been – measured via the angular distribution of two body decays of the meson in its helicity rest frame. Our results hold for small p_T and large z hadrons, in particular we expect them to hold for heavy mesons which should more easily satisfy such requirements.

Our results for ϕ, (49), are in agreement with data, $\mathrm{Re}\rho_{1,-1}(\phi) = -0.11 \pm 0.07$ [3]; notice that such data refer to values of $z > 0.7$ and $\cos\alpha = 0.9$, but still have large errors. Our results for D^*, (48), have the same negative sign, but are larger in magnitude than the value found by the OPAL collaboration, $\mathrm{Re}\rho_{1,-1}(D^*) = -0.039 \pm 0.016$ [3]. There are good reasons for that: data on D^* are collected for $z > 0.5$, and might still contain events to which our calculations do not apply and for which one expects $\rho_{1,-1} = 0$; one should also not forget that our predictions are somewhat lessened (in magnitude) by contributions from $S_x = 1$ [see comment after (33)].

We notice that while the mere fact that $\rho_{1,-1}$ differs from zero is due to a coherent fragmentation of the $q\bar{q}$ pair, the actual numerical values depend on the Standard Model coupling constants; for example, $\rho_{1,-1}$ would be positive at smaller energies, at which the one gamma exchange dominates, while it is negative at LEP energy where the one Z exchange dominates. $\rho_{1,-1}$ has also a peculiar dependence on the meson production angle, being small at small and large angles and maximum at $\theta = \pi/2$.

Such coherent effects in the fragmentation of quarks might not play a role in unpolarized observables, where they are usually neglected; however, they should be taken into account when dealing with more subtle quantities like off-diagonal spin density matrix elements. Many of these effects vanish in the limit of small intrinsic momentum of the hadron inside the jet, $p_T/E_h \to 0$; this happens, for example, in the fragmentation of quarks into spin 1/2 hadrons [2]. The quantity considered here, instead, survives also in the small p_T limit; we actually exploit such a limit in order to make numerical predictions.

The recent data [3] are encouraging; it would be interesting to have more and more detailed data, possibly with a selection of final hadrons with the required features for our results to hold. A measurement of the p_T of final hadrons and a study of the dependence of several observables on its value would offer many more possibilities of testing both the dynamics of the fragmentation process and unusual aspects of the basic interactions; a measurement of $\rho_{+-}(\Lambda)$ with a selection of Λ particles with $p_T \neq 0$ is already available [4] and in agreement with our expectations.

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